

# YbRh<sub>2</sub>Si<sub>2</sub>: Quantum tricritical behavior in itinerant electron systems

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We propose that proximity of the first-order transition manifested by the quantum tricritical point (QTCP) explains non-Fermi-liquid properties of YbRh<sub>2</sub>Si<sub>2</sub>. Here, at the QTCP, a continuous phase transition changes into first order at zero temperature. The non-Fermi-liquid behaviors of YbRh<sub>2</sub>Si<sub>2</sub> are puzzling in two aspects; diverging ferromagnetic susceptibility at the antiferromagnetic transition and unconventional power-law dependence in thermodynamic quantities. These puzzles are solved by an unconventional criticality derived from our spin fluctuation theory for the QTCP.

**KEYWORDS:** quantum critical phenomena, quantum tricritical point, YbRh<sub>2</sub>Si<sub>2</sub>, non-Fermi-liquid behavior, self-consistent renormalization theory

Critical temperatures of the symmetry-breaking phase transitions can be lowered to zero at the quantum critical point (QCP) by tuning quantum fluctuations such as by magnetic fields as shown in Fig. 1(a). Quantum critical phenomena in metals have attracted much interest from both theoretical and experimental points of view, because of not only its own right but also unconventional superconductivity as well as non-Fermi-liquid behavior observed near the QCP.<sup>1,2</sup>

The conventional spin fluctuation theory of the QCP by Moriya, Hertz and Millis<sup>2-5</sup> has succeeded in explaining a number of non-Fermi-liquid properties. However, this picture has been challenged by many recent experiments,<sup>1,2,6</sup> where criticalities of thermodynamic and transport properties do not follow it.

A typical heavy-fermion compound YbRh<sub>2</sub>Si<sub>2</sub><sup>1</sup> belongs to such an unconventional category. At the magnetic field  $H = 0$ , it exhibits an antiferromagnetic (AF) transition at the Néel temperature  $T_N = 0.07\text{K}$ . An AF QCP emerges at the critical magnetic field  $H_c \sim 0.06\text{T}$  along the  $c$  axis.<sup>7,8</sup> Near  $H_c$ , Sommerfeld coefficient of specific heat  $\gamma$  is logarithmically increased with lowering temperature  $T$  above  $0.3\text{K}$  and even faster below it<sup>9</sup> in contrast to the conventional theory predicting convergence to a constant. Transport and optical data roughly show the resistivity linearly scaled with  $T$  and frequency.<sup>7</sup> Among all, a key aspect is an unusually enhanced ferromagnetic susceptibility  $\chi_0$  roughly scaled by  $\chi_0 \propto T^{-\zeta}$  and  $\chi_0 \propto |H - H_c|^{-\zeta'}$  with  $\zeta \sim \zeta' \sim 0.6^8$  contradicting the standard expectation of saturation to a constant. In accordance, the magnetization shows convex dependence on  $H$ .<sup>8</sup> NMR<sup>10</sup> and ESR<sup>11</sup> signals are also consistent. These non-Fermi-liquid properties are all contradicting the standard theory<sup>2-5</sup> for the AF QCP and are under extensive debates.<sup>6</sup>

A hint comes from the fact that the first-order transition is observed for YbRh<sub>2</sub>Si<sub>2</sub> under pressure.<sup>12</sup> Actually, the proximity of the first-order transition is common in many compounds with unconventional non-Fermi-liquid properties. Our idea is that the proximity of the first-order transition, namely, tricriticality solves the puzzle

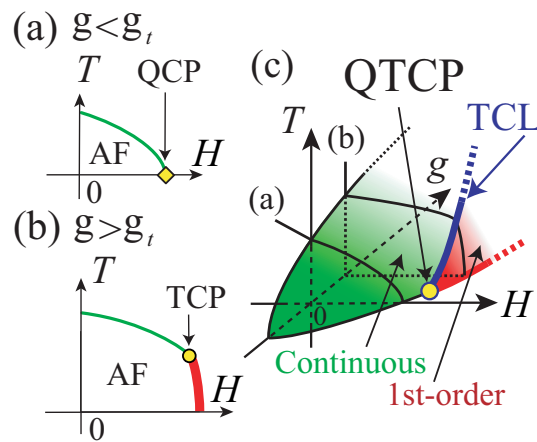


Fig. 1. (color online). (a) Phase diagram of antiferromagnetic (AF) phase with critical line [solid (green) curve] ending at the QCP [(yellow) diamond] in  $T$ - $H$  plane, where  $T$  ( $H$ ) represents temperature (magnetic field). (b) Phase diagram with TCP [(yellow) circle] separating the continuous [thin (green) curve] and first-order [thick (red) curve] transition lines. (c) Global phase diagram with tricritical line (TCL) separating the surfaces of continuous [above TCL (green)] and the first-order [below TCL (red)] surfaces. Here  $g$  represents parameter to control quantum fluctuations. In YbRh<sub>2</sub>Si<sub>2</sub>,  $g$  may correspond to pressure measured from the ambient one. The QTCP (circle) appears at  $(g, H, T) = (g_t, H_t, 0)$ , namely the endpoint of TCL. Cross sectional view at constant  $g$  for  $g < g_t$  [ $g > g_t$ ] corresponds to the phase diagram (a) [(b)].

because the tricriticality necessarily induces ferromagnetic tendency even at a clear AF transition.

At  $T \neq 0$ , the tricritical point (TCP) where phase transitions change from continuous to first order as in Fig. 1 (b) has been studied in detail.<sup>13</sup> A characteristic feature of TCP is the diverging susceptibility not only at the ordering wavenumber  $Q$  but also at zero ( $\chi_0$ ).<sup>14</sup>

If quantum fluctuations suppress the temperature of TCP to zero, *quantum tricritical point* (QTCP) appears [see Fig. 1(c)]. Then QTCP may alter the criticality of QCP as a proximity of the first-order transition.

Recently TCP has been studied for the *itinerant* ferromagnet<sup>15</sup> to understand the nature of the global phase

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diagram of weak itinerant ferromagnets  $\text{ZrZn}_2$  and  $\text{UGe}_2$ . Furthermore, ferromagnetic QTCP has been studied for itinerant helical ferromagnet  $\text{MnSi}$ <sup>16</sup> and nearly ferromagnetic metal  $\text{Sr}_3\text{Ru}_2\text{O}_7$ .<sup>17</sup> However, these previous studies on the ferromagnetic QTCP do not explain the unconventional coexistence of the ferromagnetic and AF fluctuations observed near the AF QCP in  $\text{YbRh}_2\text{Si}_2$ .

In this letter, we propose that the proximity of the first-order transition opens a way to solve the puzzles in the AF quantum critical phenomena. The proximity of the first-order transition inherently generates diverging ferromagnetic fluctuations concomitantly with the order-parameter (AF) fluctuations. The emergence of the concomitance is manifested by the *quantum tricriticality*, which generates an unexplored non-Fermi liquid. An unconventional scaling is derived by extending the self-consistent renormalization (SCR) theory<sup>3</sup> for spin fluctuations. Our result accounts for the otherwise puzzling properties of  $\text{YbRh}_2\text{Si}_2$ , even when we do not consider the possible valence transitions<sup>18</sup> or collapse of  $f$ -electron itinerancy as in the picture of the local quantum criticality.<sup>19</sup>

To understand the QTCP, we start from a standard Ginzburg-Landau-Wilson (GLW) expansion effective action for bosonic spin fields  $\varphi_q$  at the wave number  $q$ :<sup>3-5</sup>

$$S[\varphi_q] = \frac{1}{2} \sum_q r_q |\varphi_q|^2 + \sum_{q, q', q''} u(q, q', q'') (\varphi_q \cdot \varphi_{-q'}) \times (\varphi_{q''} \cdot \varphi_{q' - q - q''}) + v \sum_{q_1 \sim q_5} (\varphi_{q_1} \cdot \varphi_{-q_2}) (\varphi_{q_3} \cdot \varphi_{-q_4}) \times (\varphi_{q_5} \cdot \varphi_{q_2 + q_4 - q_1 - q_3 - q_5}) - H \varphi_0, \quad (1)$$

where  $H$  is external magnetic field;  $u(q, q', q'')$  and  $v$  are constants, while  $r_q$  depends on the magnetic field  $H$ . From eq. (1), the free energy  $F$  is obtained from

$$\exp(-F/T) = \int \prod_q \mathcal{D}\varphi_q \exp(-S[\varphi_q]/T). \quad (2)$$

Since the QTCP is expressed by fluctuations at both the AF Bragg wavenumber  $Q$  and 0, we approximate the free energy as a function of the order parameter  $M^\dagger = \langle \varphi_Q \rangle$  and the uniform magnetization  $M = \langle \varphi_0 \rangle$ :

$$F_0 = \frac{1}{2} \tilde{r}_Q M^{\dagger 2} + \tilde{u}_Q M^{\dagger 4} + v M^{\dagger 6} + \frac{1}{2} \tilde{r}_0 M^2 + \tilde{u}_0 M^4 + v M^6 - H M, \quad (3)$$

where  $\tilde{r}_Q$ ,  $\tilde{u}_Q$ ,  $\tilde{r}_0$ ,  $\tilde{u}_0$  and  $\mathcal{K}$  are defined as  $\tilde{r}_Q(T, H) = r_Q(H) + 12u_Q(\mathcal{K} + M^2) + 90v(\mathcal{K} + M^2)^2$ ,  $\tilde{u}_Q(T, H) = u_Q + 15v(\mathcal{K} + M^2)$ ,  $\tilde{r}_0(T, H) = r_0(H) + 12u_0\mathcal{K} + 90v\mathcal{K}^2$ ,  $\tilde{u}_0(T, H) = u_0 + 15v\mathcal{K}$ , and

$$\mathcal{K} = \sum_{q \neq 0, Q} \langle |\varphi_q|^2 \rangle. \quad (4)$$

Effects of spin fluctuations are included in  $\mathcal{K}$  following the SCR theory. We approximate  $u(q, q, Q)$  [ $u(q, q, 0)$ ] and the equivalent coefficients as  $q$ -independent values;  $u(q, q, Q) \simeq u_Q$  [ $u(q, q, 0) \simeq u_0$ ] for all  $q$  [for  $q \neq Q$ ].

We eliminate  $M$  in eq. (3) by using the saddle point condition for  $M$ ,  $\partial F_0 / \partial M = 0$  leading to the relation

between  $M$  and  $M^\dagger$  as

$$M = a_0 + a_1 M^{\dagger 2} + a_2 M^{\dagger 4} + \dots, \quad (5)$$

where the expansion coefficients  $a_0 \sim a_2$  are determined by substituting eq. (5) into the saddle point condition:

$$\tilde{r}_0(T, H) a_0 + 4\tilde{u}_0(T, H) a_0^3 + 6v a_0^5 - H = 0, \quad (6)$$

$$12a_0 \tilde{u}_Q(T, H) + a_1 R(T, H) = 0, \quad (7)$$

where  $R(T, H) = \tilde{r}_0(T, H) + 12\tilde{u}_0(T, H) a_0^2 + 30v a_0^4$ . By using eq. (5), we obtain the free energy as

$$F_0 = \frac{1}{2} \tilde{r}_Q(T, H) M^{\dagger 2} + \tilde{u}'_Q(T, H) M^{\dagger 4} + O(M^{\dagger 6}), \quad (8)$$

where  $\tilde{u}'_Q(T, H) = \tilde{u}_Q(T, H)(1 + 6a_0 a_1)$ . In eq. (8), continuous phase transitions occur at  $\tilde{r}_Q = 0$  when  $\tilde{u}'_Q(T, H) > 0$ , while the first-order phase transitions occur when  $\tilde{u}'_Q(T, H) < 0$ .<sup>13, 14</sup> Therefore, the QTCP appears when the conditions  $\tilde{r}_Q(0, H_t) = 0$  and  $\tilde{u}_Q(0, H_t) = 0$  are both satisfied,<sup>20</sup> where  $H_t$  is the critical field at the QTCP.

We now discuss the susceptibilities  $\chi_Q$  at the AF vector  $Q$  and  $\chi_0$  at  $q = 0$  in the disordered phase ( $M^\dagger = 0$ ,  $M = a_0$ ) by using eq. (6) and the free energy (8). From eq. (8),  $\chi_Q^{-1}$  is given as

$$\chi_Q^{-1} = \left. \frac{\partial^2 F_0}{\partial M^{\dagger 2}} \right|_{M^\dagger=0} = \tilde{r}_Q(T, H). \quad (9)$$

By differentiating eq. (6) with respect to the magnetic field  $H$ , we obtain  $\chi_0^{-1}$  as

$$\chi_0^{-1} \equiv \left( \frac{\partial a_0}{\partial H} \right)^{-1} = \frac{R(T, H)}{1 - a_0 \partial \tilde{r}_0 / \partial H - 4a_0^3 \partial \tilde{u}_0 / \partial H} \propto \tilde{u}_Q(T, H). \quad (10)$$

Here, we used eq. (7), which gives  $R(T, H) \propto \tilde{u}_Q(T, H)$ .

Now, the fluctuation-dissipation (FD) theorem<sup>21</sup>

$$\sum_{q \neq 0, Q} \langle |\varphi_q|^2 \rangle = \frac{2}{\pi} \int_0^\infty d\omega \left( \frac{1}{2} + n(\omega) \right) \sum_{q \neq 0, Q} \text{Im} \chi(q, \omega), \quad (11)$$

and  $n(\omega) \equiv 1/(e^{\omega/T} - 1)$ , combined with eqs. (4), (6), (9), and (10) constitute the self-consistent equations<sup>3</sup> to determine  $\mathcal{K}$  and  $\chi$  in the scheme of our extended SCR theory. Using this SCR theory, we now clarify how the susceptibilities and the magnetization measured from the QTCP ( $\chi_Q^{-1}$ ,  $\chi_0^{-1}$ ,  $\delta a_0 \equiv a_0 - a_{0t}$  with  $a_{0t}$  being the value at the QTCP) are scaled with  $\delta H = H - H_t$  and  $T$  near the QTCP. The results will be shown in eqs. (15)-(17).

In the SCR theory, non-trivial temperature dependence of physical properties comes from the spin fluctuation term  $\mathcal{K}$ . Therefore, we first clarify the scaling of  $\mathcal{K}$  by using the FD theorem combined with expansions of  $\chi_{0+q}(\omega)$  and  $\chi_{Q+q}(\omega)$  in terms of the wavenumber  $q$  and the frequency  $\omega$  near the QTCP. The ordering susceptibility  $\chi_{Q+q}(\omega)$  is assumed to follow the conventional Ornstein-Zernike form ( $\chi_{Q+q}(\omega)^{-1} \simeq \chi_Q^{-1} + A_Q q^2 - iC_Q \omega$ ) as in the SCR formalism, while the uniform part  $\chi_{0+q}(\omega)$  turns out *not* to follow. This is because the scaling relation  $\chi_0^{-1} \propto \chi_Q^{-1/2}$  holds near TCP within the GLW theory.<sup>13</sup> As we will see, the self-consistency among eqs. (4), (6), (9), (10), and (11) requires that this relation still holds for  $q \neq 0$ . Therefore,

we obtain the relation as  $\chi_{0+q}(0)^{-1} \propto \chi_{Q+q}(0)^{-1/2} \propto (\chi_Q^{-1} + A_Q q^2)^{1/2} \propto (\chi_0^{-2} + A_0 q^2)^{1/2}$ . From the conservation law,  $\omega$  dependence of  $\chi_{0+q}(\omega)^{-1}$  should be given as  $\chi_{0+q}(\omega)^{-1} \simeq \chi_{0+q}(0)^{-1} - iC_0\omega/q$ . Finally, we obtain  $\omega$  and  $q$  expansions of  $\chi_{0+q}(\omega)^{-1}$  as  $\chi_{0+q}(\omega)^{-1} \simeq (\chi_0^{-2} + A_0 q^2)^{1/2} - iC_0\omega/q$ .

By substituting the above forms for  $\chi_{0+q}(\omega)^{-1}$  [ $\chi_{Q+q}(\omega)^{-1}$ ] into the FD theorem (11), the contributions from the spin fluctuations near zero [ordering] wavenumber is given as  $\sum_{q \sim 0} \langle \varphi_q^2 \rangle \simeq K_{00} - K_{01}\chi_0^{-2} + K_{0T}T^2$ , [ $\sum_{q \sim Q} \langle \varphi_q^2 \rangle \simeq K_{Q0} - K_{Q1}\chi_Q^{-1} + K_{QT}T^{3/2}$ ] where  $K_{00}$ ,  $K_{01}$ , and  $K_{0T}$  [ $K_{Q0}$ ,  $K_{Q1}$ , and  $K_{QT}$ ] are constants. From these relations, in three dimensions, we obtain the scaling of  $\delta\mathcal{K}$  measured from the QTCP as

$$\delta\mathcal{K} \simeq -K_{01}\chi_0^{-2} - K_{Q1}\chi_Q^{-1} + K_{0T}T^2 + K_{QT}T^{3/2}. \quad (12)$$

The singularity of magnetization  $a_0$  is obtained by solving eq. (6). Near the QTCP, eq. (6) can be approximated as  $A\delta a_0^2 + B\delta a_0 + C = 0$ , with  $A = 12a_{0t}(5va_{0t}^2 + \tilde{u}_0)$ ,  $B = \delta\tilde{r}_0 + 12a_{0t}^2\delta\tilde{u}_0$ , and  $C = a_{0t}\delta\tilde{r}_0 + 4a_{0t}^3\delta\tilde{u}_0 - \delta H$ , where  $\delta\tilde{r}_0 = \tilde{r}_0(T, H) - \tilde{r}_0(0, H_t)$ , and  $\delta\tilde{u}_0 = \tilde{u}_0(T, H) - \tilde{u}_0(0, H_t)$ . Since both  $B$  and  $C$  vanish at the QTCP, we obtain the asymptotic behavior of  $\delta a_0$  as

$$\delta a_0 \simeq (\alpha_0\delta H + \alpha_1\delta\mathcal{K})^{1/2}, \quad (13)$$

where  $\alpha_0, \alpha_1$  are constants.

By defining  $\delta\tilde{r}_Q(T, H) \equiv \tilde{r}_Q(T, H) - \tilde{r}_Q(0, H_t)$ , we get

$$\chi_Q^{-1} = \delta\tilde{r}_Q(T, H) = \delta r_Q(H) + 90v(\delta\mathcal{K} + \delta\tilde{a}_0)^2, \quad (14)$$

since both  $\tilde{r}_Q(0, H_t)$  and  $\tilde{u}_Q(0, H_t)$  are zero at the QTCP and terms linear in  $\delta\mathcal{K}$  and  $\delta\tilde{a}_0$  vanish. Here  $\delta r_Q$  and  $\delta\tilde{a}_0$  are defined as  $\delta r_Q = r_Q(H) - r_Q(H_t) \simeq r_{QH}\delta H$ ,  $\delta\tilde{a}_0 = a_0^2 - a_{0t}^2 = \delta a_0(\delta a_0 + 2a_{0t})$ .

From eqs. (12) and (13),  $\delta\mathcal{K}$  is higher order of  $\delta a_0$  near the QTCP. Then, from eqs. (10) and (14), the most dominant terms of  $\chi_Q^{-1}$  and  $\chi_0^{-1}$  are given as  $\chi_Q^{-1} \propto \delta a_0$ ,  $\chi_Q^{-1} \simeq r_{QH}\delta H + 360va_{0t}^2\delta a_0^2$ , which together with eqs. (12) and (13) leads to  $\delta H$  and  $T$  dependence as

$$\chi_Q^{-1} \simeq \beta_{Q0}\delta H + \beta_{Q1}T^{3/2}, \quad (15)$$

$$\chi_0^{-1} \simeq (\beta_{00}\delta H + \beta_{01}T^{3/2})^{1/2}, \quad (16)$$

$$\delta a_0 \simeq (\alpha'_0\delta H + \alpha'_1T^{3/2})^{1/2}. \quad (17)$$

Singularities of the uniform susceptibility  $\chi_0$ , the ordering susceptibility  $\chi_Q$ , and the magnetization  $\delta a_0$  near the QTCP are summarized in Fig. 2. We note that the singularity of  $\chi_Q^{-1}$  given by Green *et al*<sup>17</sup> as  $T^{8/3}$  is not correct, since they neglect the  $T^2$  dependence of the bare second-order coefficient  $r_q$ .<sup>5</sup>

We now examine whether the criticality of the QTCP is consistent with the experimental results of YbRh<sub>2</sub>Si<sub>2</sub>. We first emphasize that the puzzling critical exponents in YbRh<sub>2</sub>Si<sub>2</sub> described above for  $T$  and  $H$  dependences of  $\chi_0$  and  $M$  are well consistent with the quantum tricriticality derived from eqs. (16) and (17), while any other theories do not reproduce these exponents.

To compare with the experimental results more quantitatively, we now solve the self-consistent equations numerically, and obtain the uniform susceptibility  $\chi_0$  and the magnetization  $\delta a_0$  near the QTCP as follows:

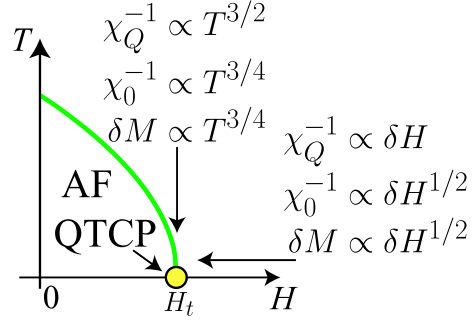


Fig. 2. (color online). Schematic phase diagram around the QTCP under the magnetic fields  $H$ .  $\delta M$  denotes the magnetization measured from the critical value at the QTCP.

We first approximate the magnetic field dependence of  $\delta r_0(H) \equiv r_0(H) - r_0(H_t)$  as  $\delta r_0(H) \simeq r_{0H}\delta H$ . The contributions from the spin fluctuations, namely  $\mathcal{K}$ , can be calculated by setting the four SCR parameters  $T_{0A}$ ,  $T_{00}$ ,  $T_{QA}$ , and  $T_{Q0}$ .<sup>22</sup> Furthermore, once the parameters  $v$ ,  $r_{QH}$ ,  $r_{0H}$ ,  $H_t$ , and  $a_{0t}$  are fixed, the other parameters ( $r_0$ ,  $r_Q$ ,  $u_0$ , and  $u_Q$ ) are determined from the conditions  $\tilde{r}_Q(0, H_t) = 0$ ,  $\tilde{u}_Q(0, H_t) = 0$  and eqs. (6), (7). In this letter, to calculate physical properties, we employ a reasonable set of parameters given in ref. 23 as follows: We estimate  $H_t$  and  $a_{0t}$  directly from experiments and also choose conventional SCR parameters ( $T_{0A}$ ,  $T_{00}$ ,  $T_{QA}$ ,  $T_{Q0}$ ) within the order of 10-100K. This range of SCR parameters is typical in heavy fermion compounds.<sup>3</sup> In contrast to these, for the other non-primary parameters ( $r_{QH}$ ,  $r_{0H}$ , and  $v$ ), we do not find any constraint from physical requirement. Therefore, we have freely tuned these parameters to reproduce the experimental results quantitatively. However, the critical exponents do not change even if we have chosen these parameters arbitrarily. Microscopic derivation of these phenomenological parameters is left for future studies.

In Fig. 3 (a), the numerical result of the temperature dependence of  $\chi_0^{-1}$  just on the QTCP is compared with the experimental  $\chi_0^{-1}$  reported in ref. 8. Although we obtain the critical exponent  $\zeta = 0.75$  ( $\chi_0^{-1} \propto T^\zeta$ ) for  $T \rightarrow 0$ , the numerical result shows that  $\chi_0^{-1}$  is roughly scaled by  $T^{0.6}$  at higher temperatures ( $T > 1.0$ K). We emphasize that the puzzling convex behavior of  $\chi_0^{-1}$  for YbRh<sub>2</sub>(Si<sub>0.95</sub>Ge<sub>0.05</sub>)<sub>2</sub> near the QCP can not be accounted for by the conventional quantum criticality, because the critical exponent  $\zeta$  is always larger than one for the conventional quantum critical point.<sup>3-5</sup> The nonzero offset of experimental  $\chi_0^{-1}$  at  $T = 0$  indicates that the QCP in YbRh<sub>2</sub>(Si<sub>0.95</sub>Ge<sub>0.05</sub>)<sub>2</sub> exists slightly away from the QTCP. It is an intriguing experimental challenge to determine the precise location of the QTCP by tuning the pressure and the magnetic field.

In Fig. 3 (b), the numerical result of magnetization curve at  $T = 0$  is compared with the experimental result reported in ref. 8. By using the same parameters, it is noteworthy that not only the uniform susceptibility but also the magnetization is consistent with the experimental result quantitatively in the relevant parameter region. Because the experimentally observed QCP is

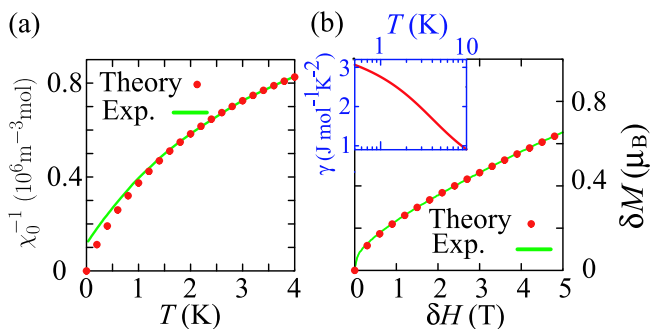


Fig. 3. (color online). (a) Experimental  $\chi_0^{-1}$  for  $\text{YbRh}_2(\text{Si}_{0.95}\text{Ge}_{0.05})_2$  at  $H = 0.03(\text{T})$  reported in ref. 8 compared with the present SCR theory. Green line (red circle) represents the experimental (theoretical)  $\chi_0^{-1}$ . Theoretical  $\chi_0^{-1}$  is calculated just on the QTCP ( $H = H_t$ ). (b) Experimental magnetization curve for  $\text{YbRh}_2(\text{Si}_{0.95}\text{Ge}_{0.05})_2$  at  $T = 0.09(\text{K})$  reported in ref. 8 compared with the present theory. Green line (red circle) represents the experimental (theoretical) magnetization curve.  $\delta M$  ( $\delta H$ ) represents the magnetization (magnetic field) measured from the critical value. We estimate the experimental critical magnetic field  $H_c$  (magnetization  $M_c$ ) as  $0.027(\text{T})$  ( $0.004 (\mu_B)$ ). The inset shows the numerical result of Sommerfeld coefficient of the specific heat  $\gamma$  just on the QTCP.

slightly away from the QTCP, small deviations are seen at low temperatures and low magnetic fields ( $T < 1.0\text{K}$  and  $\delta H < 0.5\text{T}$ ). We note that this singularity of the magnetization [ $\delta M \propto \delta H^{1/2}$  as seen from eq. (17) and Fig. 3 (b)] is qualitatively different from that of the conventional metamagnetic transitions. It has been proposed that they belong to the Ising universality class.<sup>24</sup> The critical exponent  $\delta$  ( $\delta M \propto \delta H^{1/\delta}$ ) of the Ising universality is always larger than three at any dimensions. The present critical exponent  $\delta = 2$  makes a sharp contrast to such conventional critical exponents  $\delta \geq 3$ .

We now discuss the singularity of the Sommerfeld coefficient of the specific heat  $\gamma$ . In the inset of Fig. 3 (b), the numerical result of  $\gamma$  just on the QTCP is shown. At high temperatures ( $T > 1.0\text{K}$ ), both the singularity ( $\gamma \propto -\log T$ ) and the amplitude are consistent with those of the experimental result.<sup>9</sup> However, at low temperatures ( $T < 1.0\text{K}$ ), within this SCR theory,  $\gamma$  near the AF QTCP approaches that of the conventional AF QCP ( $\gamma \propto \text{const.} - T^{1/2}$ ), while experimentally, power-law-like behavior is observed for  $T < 0.3\text{K}$ .<sup>9</sup> This discrepancy may be solved by considering either the fact that the Néel temperature is actually nonzero or effects of valence fluctuations,<sup>18</sup> while the criticality of magnetic properties analyzed here should remain unchanged.

Finally, we discuss a different scenario for the QCP in  $\text{YbRh}_2\text{Si}_2$  proposed by Coleman *et al.*<sup>19</sup> They claim that a breakdown of composite heavy fermion (namely, all  $f$  electrons decouple from the Fermi surface) occurs at the QCP and no heavy electron exists in the ordered phase any more. This scenario is inconsistent with a large Sommerfeld coefficient of the specific heat  $\gamma$  even in the ordered phase.<sup>9</sup> While a large change of Hall constant in  $\text{YbRh}_2\text{Si}_2$ <sup>25</sup> was suggested to support their scenario, Norman<sup>26</sup> has pointed out that small changes of the  $f$

electron occupation are sufficient to reproduce the experimental result by calculating the band structures of  $\text{YbRh}_2\text{Si}_2$ . Steep change in the Hall coefficient is then naturally understood under the proximity of the first-order transition.

In summary, a non-Fermi liquid different from that obtained from the conventional QCP is shown to emerge when the proximity of the first-order transition is involved through the QTCP. The unconventional criticality thus obtained by the extension of the SCR theory solves the puzzles in the experimental results of  $\text{YbRh}_2\text{Si}_2$ . It is intriguing to examine whether this proximity also plays roles in other unconventional non-Fermi liquids.

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